Dispersed Information, Social Networks and Aggregate Fluctuations

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Abstract

This paper argues that dispersed information may explain aggregate fluctuations. When agents have incomplete and heterogeneous information, it is optimal to consider the actions of other agents because of the additional information conveyed by these actions. We call the act of using other agents' actions in the individual decision process social learning. This paper shows that social learning aimed at increasing the precision of individual information may lead to aggregate fluctuations. We consider a setting where firms receive independent noisy signals about a common fundamental and can observe other firms' actions through a network of informational links. We show that, when firms can observe each other's decisions, they increase the accuracy of their actions. While reducing volatility at the individual level, social learning may lead to an increase in volatility at the aggregate level depending on the network topology. Moreover, if the network is very asymmetric, aggregate volatility does not decay as predicted by the law of large numbers.

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1 Introduction

This paper proposes a simple microfoundation for aggregate fluctuations. It suggests that aggregate fluctuations may originate at the micro level in the presence of informational externalities among agents.

Human beings are social animals. This simple truth is often neglected when analyzing human behavior in macroeconomics. The characteristic of social animals is that they live in groups and interact with other members of the group to perform vital tasks, such as defense or feeding. Several studies in biology document forms of interaction in which the group allows individuals to take advantage of the information gathered by others. Pulliam (1973) for example studies the flocking behavior of finches and shows that an advantage of feeding in groups is to increase the probability to detect a predator ("many eyes effect"). If one finch spots a predator and decides to fly off in alarm, the other finches observe this action and follow without having actually seen the danger. Moreover, assuming that anti-predatory vigilance is costly, for example because it is time consuming and alternative to feeding, the group provides a simple and effective cost sharing mechanism (Fernandez et al., 2003). In fact, the propensity to live in groups and learn from others' behavior can be considered an evolutionary response that promotes survival in complex environments for both animals and human beings (Henrich, 2015).

There are countless social and economic situations in which human beings are influenced by what others around them are doing when deciding upon an action. The decisions of others can be relevant for individual decision making for a variety of reasons. First, in the presence of *payoff externalities*, the actions of others can directly influence the utility function. Typical examples are market environments with strategic complementarities. Second, the decisions of other agents may matter for individual decision making in the presence of *informational externalities*. In such environments individual decisions reflect relevant information, hence observing the actions of others allows to exploit such information.

We focus on the second scenario and consider the case in which informational externalities arise due to *dispersed information*, i.e., information which is incomplete and heterogeneous across agents. Following Bandura and McClelland (1977), we call the act of observing and learning from other agents' actions *social learning*.

The goal of this paper is to show that, in the presence of dispersed information, aggregate fluctuations can arise as a consequence of social learning depending on the structure of the social network shaping the patterns of interaction and learning among individuals. To this end, we consider a setting where agents receive independent noisy signals about the true value of a common variable of interest and social learning occurs through an arbitrary social network. Although in our setting individual payoffs do not depend on the decisions of others, paying attention to other individuals is rational because their decisions reflect valuable additional information. This simple setup, in which there are no strategic complementarities and the interaction among agents is purely informational, allows us to isolate the impact of different network topologies on the transmission of idiosyncratic noise from the individual to the aggregate level.[1]

The essence of the model is as follows. A set of agents must take an action, receive a private signal on a common payoff-relevant fundamental state variable, and can observe the actions of a subset of other agents. The subset of observable agents is defined by a social network. The "observational structure" defined by the network is exogenous and can correspond to e.g., geographical proximity or social relationships. The setup of the model is very general and can be applied to a variety of economic environments. For the sake of concreteness, we frame the analysis in a setup where firms must decide the investment level before they can observe the true return on capital. They receive a private signal on the common productivity and they can observe the investment decisions of a subset of other firms. In this setting we show that social learning aimed at increasing the precision of individual information may lead to aggregate fluctuations.[2] Our main results, contained in Propositions 1 – 3 below, can be summarized as follows. First, the case of isolated firms, i.e., firms acting only in reaction to their own signal, represents an upper bound for the volatility of individual investment decision. In fact, when firms can observe each other's decisions, they are able to increase the accuracy of their investment plans. This is quite intuitive as, with social learning, each firm is able to exploit the additional information embedded

[1]Models with noisy signals and strategic complementarities have been considered by Morris and Shin (2002), Angeletos and Pavan (2007), Angeletos and La'O (2013), Colombo et al. (2014), Benhabib et al. (2015), Chahrour and Gaballo (2015) and Angeletos et al. (2016) among others. All papers mentioned above abstract from considerations about the macroeconomic impact of social learning for different social networks topologies, which is instead the focus of this paper.

[2]We frame our analysis in an investment decision model due to its simplicity. In Appendix C we analyze the effect of social learning in a slightly more general macroeconomic setting featuring demand and supply. In particular, we consider a Lucas-Phelps island economy where firms optimally set prices.

in other firms' decisions. Moreover, this result is independent on the topology of the network.

Second, while reducing volatility at the individual level, social learning may, on the other hand, lead to an increase in volatility at the aggregate level. In fact, the case of isolated firms represents a lower bound for the volatility of aggregate investment and, according to the properties of the network topology, learning by observing other firms' behavior can imply a higher variance of aggregate investment.

Finally, our paper also contributes to the debate on whether significant aggregate fluctuations may originate from idiosyncratic shocks at the micro level. This possibility has traditionally been discarded following the standard diversification argument. According to the latter, in an economy composed by N firms, the aggregate effect of independent idiosyncratic disturbances should decay at a rate $1/\sqrt{N}$ and it is therefore negligible in large economies (see e.g., Lucas, 1977). We show that the diversification argument may not hold in the presence of dispersed information and social learning. Social networks may translate imperfect information into volatility at the aggregate level. In particular, our results demonstrate that when the informational network is very asymmetric, then aggregate volatility decays at a rate much slower than $1/\sqrt{N}$. In other words when many firms look at the decision of the same small number of firms, then the influence exerted by firms that are very *central* in the informational network decays very slowly as the number of firms in the economy increases.

Our work relates to several strands of research. Banerjee (1992), Bikhchandani et al. (1992) and Smith and Sørensen (2000) among others study learning models in which agents can observe the actions of other agents. These papers investigate whether sequential learning mechanisms, defined as observational learning, lead to informational cascades, and to an inefficient aggregation of private information. Informational cascades are defined in Bikhchandani et al. (1992) as situations in which agents follow the actions of the preceding agents, disregarding their own private information. Informational cascades emerge when the set of possible actions is discrete (Bikhchandani et al., 1998). In our analysis we consider simultaneous decisions as in Gale and Kariv (2003), and a continuous set of possible actions, ruling out the possibility of informational cascades. Moreover, we introduce a network structure defining the patterns of social learning and show that it plays an important role for aggregate outcomes. Ellison and Fudenberg (1993, 1995) study private information aggregation when agents can observe choices and payoffs of other agents and use rule of thumb heuristics to decide their own action. They show that even with simple heuristic behaviors, social learning can lead to efficient outcomes. We analyze instead a framework in which optimizing agents do not observe other agents' payoffs and we focus on the impact of the observational network on individual and aggregate volatility.

The papers stemming from the seminal contribution of DeGroot (1974), e.g., Bala and Goyal (1998), Golub and Jackson (2010), Acemoglu et al. (2011) and DeMarzo et al. (2003), analyze instead richer network structures. These papers focus on network topologies ensuring convergence to the true underlying fundamental (or unidimensional opinions) both under Bayesian and non-Bayesian learning/updating of beliefs. We consider a setting in which agents aggregate information optimally conditional on their knowledge of the network structure as described in Section 2, and we focus on aggregate volatility.

Another important stream of literature related to our work concerns the social value of public information in presence of imperfect private information (see e.g., Morris and Shin, 2002; Angeletos and Pavan, 2004; Colombo et al., 2014, among others). In particular, the model in which we frame our analysis is similar to the model described in Angeletos and Pavan (2004), but with several substantial differences. First, the focus of our paper is on the impact of social learning on aggregate volatility rather then on social welfare. Second, in our framework agents do not have access to public information, but they can observe the actions of different subsets of other agents. Finally, to isolate the network effect on the individual actions, we assume that there are no strategic complementarities.[3]

Our work is also related to the literature on the role of idiosyncratic shocks at the micro level in macroeconomic fluctuations. Dupor (1999) and Horvath (1998, 2000) debated about the diversification argument mentioned above. Gabaix (2011) shows that the $1/\sqrt{N}$ diversification argument does not apply when the firm size distribution is sufficiently fat-tailed, while Acemoglu et al. (2012) show that the argument is not valid in the presence of asymmetric input-output links between sectors.[4] We show

[3]In the terms of the model described in Angeletos and Pavan (2004), we assume that the individual return to investment does not depend on the aggregate level of investment.

[4]Earlier contributions on the topic include Jovanovic (1987) and Durlauf (1993) who show that strategic complementarities and local firms' interactions may translate shocks occurring at the firm-level into aggregate volatility. Moreover, Bak et al. (1993) focus on the role of supply chains in

that, even in the presence of firms with identical size and without input-output relations between different sectors, the diversification argument may fail in the presence of dispersed information and social learning.

Finally, this paper is similar in spirit to Barrdear (2014), who studies the influence of social learning over an opaque observational network. Barrdear (2014) considers agents interacting in an environment featuring strategic complementarities and focuses on the impact of higher-order beliefs. In such setup, in order to make the problem tractable, it is necessary to impose some restrictions on the network structure and on agents' knowledge of the latter. We consider instead a simpler setup without strategic complementarities, which allows us to isolate the effect of arbitrary network topologies on aggregate fluctuations, without imposing any restriction on agents' knowledge of the network.

The outline of the paper is as follows. Section 2 presents a model with dispersed information and social learning. Section 3 analyzes the effect of different network topologies on volatility at the individual and aggregate level. Section 4 concludes.

2 Model

The economy is populated by a finite set of firms, $\mathcal{N} = \{1, 2, \ldots, N\}$, indexed by $i = 1 \ldots N$. Preferences and technologies are modeled as in Angeletos and Pavan (2004). In particular, firms are risk-neutral with utility

$$u_i = \theta k_i - \frac{1}{2} k_i^2 \,, \tag{1}$$

where $\theta \in \mathbb{R}$ is the return to investment, $k_i \in \mathbb{R}$ is the investment decision and $k_i^2/2$ is the cost of investment. The exogenous random variable θ parameterizes the fundamentals of the economy and is assumed to be drawn from an improper uniform distribution (Morris and Shin, 2002).

Differently from Angeletos and Pavan (2004), in our framework the return on capital does not depend on the investment decision of the other agents in the economy. This implies that there are no strategic complementarities between investment decisions. The fundamental θ is unknown when investment decisions are made so that each agent chooses k_i to maximize $\mathrm{E}_i[u_i]$. This implies that the optimal investment

aggregate fluctuations.

of firm i is

$$k_i = \mathrm{E}_i[\theta] = \mathrm{E}[\theta|\mathcal{I}_i] \,, \tag{2}$$

where $\mathcal{I}_i$ is the information set available to agent i. Firms receive a private signal s_i on the fundamental

$$s_i = \theta + \sigma\varepsilon_i \,, \tag{3}$$

where σ is the standard deviation of the private signal and $\varepsilon_i \sim \mathcal{N}(0,1)$ is an i.i.d. disturbance. Eqs. (2) - (3) ensure that firms' payoffs are independent from each other and that interaction among firms is purely informational.

When firms cannot observe investment decisions of other firms in the economy, the information set of each firm consists only of its private signal. Therefore in isolation we have that

$$k_i = \mathrm{E}[\theta|\mathcal{I}_i] = s_i \,. \tag{4}$$

Now consider the case in which firms can observe the actions of a subset of other firms (as in e.g., Banerjee, 1992; Bikhchandani et al., 1992) through an exogenous directed social network. The network is described by an $N \times N$ matrix Ψ, whose elements are $\psi_{ij} \in \{0,1\}$. If firm i observes firm j, then $\psi_{ij} = 1$, otherwise $\psi_{ij} = 0$. Matrix Ψ can be asymmetric and links can be one-sided, so that we may have $\psi_{ij} = 1$ and $\psi_{ji} = 0$. The network topology determines the observational structure, i.e., the subset of other firms observable by each firm. Moreover, we set the elements on the main diagonal of Ψ equal to zero, i.e., $\psi_{ii} = 0$, meaning that, quite naturally, firms do not need to observe themselves. We denote the subset of firms observed by firm i as $\Psi(i) = \{j \in \mathcal{N} | \psi_{ij} = 1\}$ and the information set of firm i as $\mathcal{I}_i = \{s_i, k_{j\in\Psi(i)}\}$. Given that private signals are normally distributed and, as shown below in Eq. (7), equilibrium decisions are linear aggregation of signals, optimal information weighting strategies are linear. Therefore, firm i's expected value of θ and optimal investment decision can be written as:

$$k_i = \delta_{ii}s_i + \sum_{j\in\Psi(i)} \delta_{ij}k_j \,, \tag{5}$$

where δ_{ii} is the weight assigned to the private signal s_i and δ_{ij} is the weight assigned by firm i to the investment decision of observed firm j. Firms set the values δ_{i*} optimally using available information, therefore the following necessary conditions for optimality must hold:

a) $\delta_{ii} > 0 \; \forall i$;

b) $\delta_{ij} \geq 0$ with $j \in \Psi(i) \; \forall i$;

c) $\delta_{ii} + \sum_{j \in \Psi(i)} \delta_{ij} = 1 \; \forall i$.

The first condition simply states that optimizing firms do not disregard their private signal. The second condition means that firms must weight the information contained in other firms' decisions, properly taking into account possible repetitions of information. This means that optimal information weighting should avoid the *persuasion bias*, i.e., failure to account for possible repetitions in received information as defined in DeMarzo et al. (2003).[5] The last condition ensures the unbiasedness of equilibrium signal aggregation. For notational convenience we rewrite Eq. (5) as

$$k_i = \alpha_i s_i + (1 - \alpha_i) \sum_j w_{ij} k_j \; , \qquad (6)$$

where $\alpha_i \equiv \delta_{ii}$ and $(1 - \alpha_i) w_{ij} \equiv \delta_{ij}$. Eq. (6) has an intuitive interpretation, as it distinguishes between the weight α_i given to private information relative to the information obtained from the network, and the set $\{w_{ij}\}$ representing the relative weights assigned to the decisions of observed firms. Conditions a), b) and c) can be translated into conditions on α_i and w_{ij}. In particular, necessary optimality conditions imply that

a′) $0 < \alpha_i \leq 1 \; \forall i$;

b′) $w_{ij} \geq 0$ with $j \in \Psi(i) \; \forall i$;

c′) $\sum_{j \in \Psi(i)} w_{ij} = 1 \; \forall i$.

In each period firms choose actions simultaneously. Let k denote the $N \times 1$ vector of individual investment decisions, s the $N \times 1$ vector of private signals, D the diagonal

[5] In general, given the network structure, it is not always possible to completely eliminate the persuasion bias. As argued below, our results are valid even if agents fail to properly account for possible repetitions of the information they receive and weight information according to their *subjective* relative precision.

matrix defined as $[\mathrm{D}]_{ii} = \alpha_i\ ,\forall i$, and W the $N \times N$ stochastic matrix of weights w_{ij} (i.e., $\sum_j w_{ij} = 1$). We can then write Eq. (6) in matrix form as

$$k = \mathrm{D}s + (\mathrm{I} - \mathrm{D})\mathrm{W}k\ ,$$

where the entries of matrices D and W depend on the exogenous network structure and on the weights chosen by firms. The *rational expectations equilibrium* is then given by

$$k = [\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}]^{-1}\mathrm{D}s\ . \tag{7}$$

Eq. (7) shows that, when firms' actions are observable through the informational network, the equilibrium investment decisions are linear combinations of private signals. The extreme case in which $\alpha_i = 1\ \forall i$ is equivalent to the case of isolation in which firms only consider their private information and the network does not play any role. The following lemma establishes conditions for the existence of the equilibrium in Eq. (7).

Lemma 1. *When conditions a′), b′) and c′) are satisfied, the rational expectations equilibrium in* (7) *exists for any network topology.*

The proof is in Appendix A, where we show that the equilibrium in Eq. (7) exists also under more general conditions regarding the weights α_i, i.e., when some $\alpha_i = 0$, provided that each firm in the network is reached, directly or indirectly, by at least one signal.

A discussion about the specific values of weights $\{\alpha_i, w_{ij}\}$ is now warranted. The specific values of $\{\alpha_i, w_{ij}\}$ are endogenous to the optimal information weighting problem and depend on the exogenous network structure. In order to assess the *objective* informational content of each observed decision, and set α_i and w_{ij} accordingly, firms must know the exact structure of the network. The reason is that each individual decision is an aggregation of signals, as described in Eq. (7). In fact, the information conveyed by the investment decision of each firm j, depends not only on its signal s_j, but also on the information (i.e., signals) contained in the decisions observed by firm j and so on. Hence, to weight their private information and the decisions of others according to *objective* informational content, firms need to know the source of all the information that influenced, both directly and indirectly, the decisions of other firms. This is consistent with the behavior of optimizing agents setting the weights by backward induction, knowing the equilibrium in Eq. (7). At this point it is important to remark that all the results derived in the paper are valid for *any* set of weights $\{\alpha_i, w_{ij}\}$ provided that the necessary conditions a′), b′), c′) mentioned above are satisfied. Therefore, our results apply also to cases in which information is weighted according to *subjective* (instead of *objective*) relative precisions.[6]

The equilibrium in Eq. (7) is reminiscent of the equilibrium in Acemoglu et al. (2012) but with the following important differences. First, Eq. (7) is derived in a dispersed information setup rather than an input-output multisectoral model, leading to different interpretations of structural parameters and economic results. Second, while Acemoglu et al. (2012) consider the case of homogeneous α, representing the labor share in the production technology, we analyze the case of heterogeneous α_i, representing individual information weighting.

At this point it is also worth emphasizing the differences between the information weighting *à la* DeGroot (1974) (see e.g., DeMarzo et al., 2003; Golub and Jackson, 2010, among others) and the information weighting in Eq. (6). In DeGroot (1974) agents take their decisions in each period using a weighted average of previous period's decisions, including their own. In our setting, firms take their decision using a weighted average of other firms decisions and their own private signal. The former mechanism may lead to solutions in which firms do not take into account their own signals in equilibrium. On the other hand, the mechanism described in Eq. (6) with optimizing firms, i.e., $\alpha_i > 0 \ \forall i$, ensures that the information contained in private signals is not disregarded in equilibrium decisions.

Before proceeding to the analysis of the impact of social learning on volatility, we define aggregate investment as the sum of individual investment decisions, normalized by the number of agents in the economic system:

$$K = \frac{1}{N} \sum_i k_i \ . \tag{8}$$

In the following section we present the main results of the paper, namely that in the presence of dispersed information, social learning increases the precision of firms' investment decisions (reducing therefore the variance of individual investments with respect to isolation), and that at the same time the social learning process leads to increased volatility at the aggregate level (increasing therefore the variance

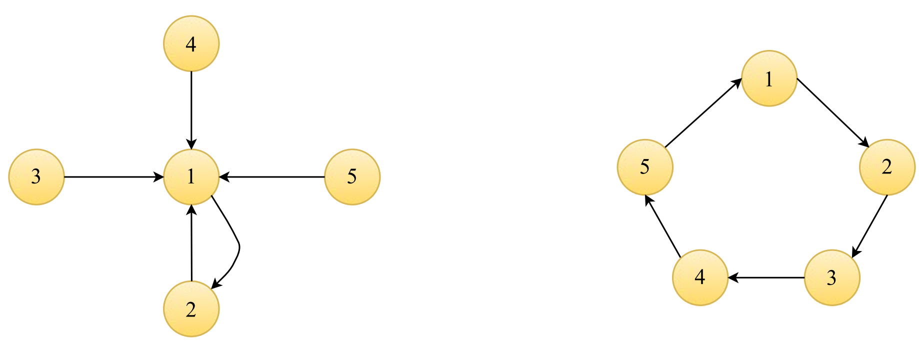
[6]In this cases the equilibrium in Eq. (7) can be thought of as the fixed point of a recursive process of decisions' updating as discussed in Appendix B.

of aggregate investment with respect to isolation) depending on the topology of the informational network. Moreover, we also characterize how the impact of firm-level independent noise on aggregate volatility decays as the number of firms $N \to \infty$, and show that social learning may lead to a decay rate slower than $1/\sqrt{N}$ depending on the network topology.

3 Network Topology and Micro/Macro Volatility

In this section we focus on the impact of social learning on both micro and macro volatility, and relate it to the structure of the network characterizing the interaction patterns among firms.

To help the intuition we will complement the exposition of our results with simple examples. Fig. 1 displays two different configurations of the economy. In both cases we have $N = 5$ and all N firms observe the actions of another firm in the economy. In the economy depicted in Fig. 1(a) as a directed star network, all firms observe the same firm $i = 1$ and the latter observes another firm $j = 2$. In the economy depicted in Fig. 1(b) as a directed regular network, each firm observes the investment decision of a different firm.



(a) Star network. (b) Regular network.

Figure 1: Different configurations of the economy.

The corresponding adjacency matrices representing the star and the regular networks are shown in Eq. (9)

$$\mathrm{W}_{star} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \mathrm{W}_{regular} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} . \tag{9}$$

In both cases, given that each firm can observe only another firm in the economy, the weight assigned to that observation is equal to 1 by construction. In the following sections we will use these simple network configurations as examples to illustrate the impact of different topologies on the micro and macro properties of the economic system, but we remark that our results are derived for arbitrary network structures.

3.1 Volatility of Individual Investment

In what follows we show that incorporating information from other firms in investment decisions is rational from a generic firm i's point of view, as social learning leads to lower variance of individual investment. Using Eq. (7), individual investment in equilibrium can be written as

$$k_i = \sum_j \hat{w}_{ij} \alpha_j s_j ,$$

where $\hat{w}_{ij}$ denotes the element (i, j) of the matrix $\hat{\mathrm{W}}$ defined as $\hat{\mathrm{W}} \equiv [\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}]^{-1}$. The variance of k_i is then given by

$$\mathrm{var}\,(k_i) = \sum_j \hat{w}_{ij}^2 \alpha_j^2 \sigma^2 . \tag{10}$$

In the case of isolated firms there are no informational links between firms, i.e., W is a matrix of zeros (hence $\hat{\mathrm{W}}$ is an identity matrix), and $\alpha_i = 1\ \forall i$, meaning that firms only consider their private signal and therefore

$$\mathrm{var}\,(k_i) = \sigma^2 . \tag{11}$$

Social learning happens when $\alpha_i < 1$ and $w_{ij} > 0$ for at least one $j \neq i$, which means that firm i has at least one informational link with another firm j in the economy, and uses the information embedded in the decision made by firm j. The impact of social learning on individual volatility is stated in the following proposition:

Proposition 1. *The variance of individual investment in the case of social learning is always less than the variance of individual investment in case of isolated firms, that is*

$$\sum_j \hat{w}_{ij}^2 \alpha_j^2 < 1 \, . \tag{12}$$

The proof is in Appendix A. Proposition 1 shows that social learning is rational from firms' point of view. In this way firms are in fact able to increase the precision of their individual forecasts of the fundamental. The intuition for this result is that the information about fundamentals contained in individual signals is spread through the network and in equilibrium firms are able to exploit this additional information by observing the behavior of other firms.

The following examples illustrate the information spreading mechanism. Before proceeding, we remark that the reasoning followed in the simple examples below to set weights $\{\alpha_i, w_{ij}\}$ reflects information weighting according to objective relative precisions. As discussed in Section 2, qualitative results do not change when information is weighted according to generic subjective relative precisions.

Example 1.

Consider an economy with $N = 5$ firms. In the absence of informational links we have that each firm sets k_i in reaction to its own signal only, i.e., $k_i = s_i$, and therefore the variance of individual investment is given by σ^2 defined as before.

Suppose instead that the informational structure of the economy is described by the regular network in Fig. 1(b). In this case the investment decision of e.g., firm 1 is taken according to $k_1 = \alpha_1 s_1 + (1 - \alpha_1) w_{12} k_2$, i.e., using both the private signal s_1 and the decision of firm 2, which in turn is taken using its private signal s_2 and the decision of firm 3, and so on. The network structure allows to incorporate information from other firms' signals in individual decisions. In fact, in equilibrium we have that investment decisions are determined according to Eq. (7) and therefore the way in which private signals are spread through the network depends on the matrix $\mathrm{C} \equiv [\mathrm{I} - (\mathrm{I} - \mathrm{D})\,\mathrm{W}]^{-1}\,\mathrm{D}$. Given the structure of the network, each firm observes directly or indirectly the decisions of all other firms in the network. Therefore, for each firm i, the information conveyed by the network already includes its own signal s_i. Consider for example firm 1 who is observing the decision of firm 2, which in turn observes the decision of firm 3 and so on. The structure of the network implies that firm 1 is observing indirectly firm 5, which is in turn observing firm 1 itself. Thus, the information contained in the signal of firm 1 is included in firm 1's decision both directly, with weight α_1, and indirectly, with weight $1-\alpha_1$, by observing the decision of firm 2. For each $\alpha_i > 0$, we have that firm i suffers from persuasion bias, in the sense that firm i fails to account for the repetition of the information contained in s_i. If the information conveyed by the individual signal s_i is already present in its observational network, each firm i should set $\alpha_i \to 0$ in order to minimize the persuasion bias and avoid overweighting of the information contained in s_i.[7]

Matrix $\mathrm{C}_{regular} \equiv [\mathrm{I} - (\mathrm{I} - \mathrm{D})\,\mathrm{W}_{regular}]^{-1}\,\mathrm{D}$ is therefore given by

$$\mathrm{C}_{regular} = \begin{pmatrix} 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \end{pmatrix} .$$

The expression above shows that, due to the network of observational links, the individual investment decision of each firm is influenced by the private signals of all firms in the economy. For example, the decision k_1 of firm 1 is given by

$$k_1 = \sum_{j=1}^{5} c_{1j} s_j = 0.2s_1 + 0.2s_2 + 0.2s_3 + 0.2s_4 + 0.2s_5 .$$

Given the topology of the regular network we have that, for each firm, the variance of individual investment with social learning is given by $0.2\sigma^2$ and thus smaller than the case of isolation.[8] Social learning allows to

[7]Notice though that α_i should be strictly positive, otherwise the information contained in s_i would not enter the network.

[8]Notice that matrix $\mathrm{C}_{regular}$ reflects optimal information weighting since in equilibrium each signal is weighted according to its objective relative precision.

exploit the information contained in individual actions and thus reduces the errors in individual investment decisions. ♣

The reduction in individual volatility in the presence of social learning with respect to the case of isolation is independent of the network structure. In fact, as long as a firm has at least one informational link and therefore looks at the decision of at least one additional firm (on top of reacting to its own signal), the variance of its investment will be lower than the case of isolation.

Nevertheless, different network topologies may imply different levels of individual volatility, as shown in the following example.

Example 2.

Consider an economy with an informational structure described by the star network in Fig. 1(a). In this case we have that firms 3, 4 and 5 are not observed by any other firm in the network. Therefore, if information weighting reflects objective relative precisions, the weights assigned by these firms to their own signal should be equal to the inverse of the total number of signals affecting their equilibrium investment decisions (the objective relative precision). Given that firms 3, 4, and 5 observe, directly and indirectly their own signal on top of the signals of firms 1 and 2, we have that $\alpha_3 = \alpha_4 = \alpha_5 = 1/3$. Instead, for both firms 1 and 2, the information contained in their private signal also appears in the information coming from the network due to the fact that these firms are observed by other firms present in their own social network (i.e., $2 \in \Psi(1)$ and $1 \in \Psi(2)$). Therefore, in order to avoid the persuasion bias, firms 1 and 2 should set $\alpha_1 = \alpha_2 \to 0$. Matrix C_{star} is therefore equal to

$$\mathrm{C}_{star} = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 1/3 & 1/3 & 0 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \end{pmatrix} \quad .$$

The expression above shows that the decisions of firms 3, 4 and 5 are based on information contained in the individual signals of two additional firms, while the decisions of firms 1 and 2 only exploit information from one additional firm. We have therefore that $\mathrm{var}(k_1) = \mathrm{var}(k_2) = 0.5\sigma^2$, while

$\text{var}(k_3) = \text{var}(k_4) = \text{var}(k_5) = 0.33\sigma^2$, meaning that individual variances are higher than in the case of a regular network where each individual decision exploits information from all the firms in the economy, but still lower than in the case of isolated firms. ♣

3.2 Volatility of Aggregate Investment

In this section we show that when firms look at other firms' decisions to make their own investment plans, aggregate variance may increase with respect to the case of isolation depending on the topology of the informational network. We denote aggregate investment normalized by the number of agents in the economy as

$$\begin{aligned} K &= \frac{1}{N}\sum_i k_i \\ &= \frac{1}{N}\sum_i \sum_j \hat{w}_{ij}\alpha_j s_j \ , \end{aligned} \tag{13}$$

where again $\hat{w}_{ij}$ denotes the element (i, j) of the matrix $\hat{\mathrm{W}}$ defined as $\hat{\mathrm{W}} \equiv [\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}]^{-1}$. Using matrix $\mathrm{C} \equiv \hat{\mathrm{W}}\mathrm{D}$ we can define the $1 \times N$ vector v' as

$$v' \equiv e'\mathrm{C} \ , \tag{14}$$

where $e' \equiv [1, \dots, 1]$, so that $v_j = \sum_i c_{ij} = \alpha_j \sum_i \hat{w}_{ij}$ and

$$K = \frac{1}{N}\sum_j v_j s_j \ .$$

Vector v can be defined as an *influence vector*, since each element v_j determines the influence of signal s_j on aggregate investment. The influence vector v is related to the Bonacich (in-degree) centrality measure (Bonacich, 1987) and it is reminiscent of the influence vector described in Acemoglu et al. (2012), with the crucial difference that the impact of firm j's signal in this case also depends on α_j. If the Bonacich centrality of firm j (summarized by the term $\sum_i \hat{w}_{ij}$) in the observational network increases, the influence of firm j's signal will increase. But there is also a second effect. Given the observational network, increasing α_j (holding constant all $\alpha_{i \neq j}$), will increase the influence of firm j's signal. The intuition is that, if α_j is relatively high, firm j's signal will be largely reflected in its investment decision, and therefore it will have relatively higher influence on the decisions of firms observing j. The variance of K can be written as

$$\text{var}\,(K) = \frac{1}{N^2} \sum_j v_j^2 \sigma^2 \ . \tag{15}$$

In the absence of social learning we have that $v_j = 1$ for all j, and therefore the variance of aggregate investment is

$$\text{var}(K) = \frac{\sigma^2}{N} \ . \tag{16}$$

The impact of social learning on aggregate volatility is described in the following proposition:

Proposition 2. *The variance of aggregate investment in the case of social learning is always greater than, or equal to, the variance of aggregate investment in the case of isolated firms, that is*

$$\frac{1}{N} \sum_j v_j^2 \geq 1 \ . \tag{17}$$

The proof is in Appendix A. According to Proposition 2, the case of isolated firms represents a lower bound for aggregate volatility. The intuition for this result is that social learning introduces correlation among individual decisions. The variance of aggregate investment depends on $\text{var}(\sum_i k_i)$, i.e., the variance of the sum of individual investment decisions. In the absence of social learning we have that individual decisions are independent from each other and therefore the aggregate variance is simply given by the sum of the variances of individual decisions, that is

$$\text{var}\left(\sum_i k_i\right) = \sum_i \text{var}(k_i) \ . \tag{18}$$

In the presence of social learning, individual decisions are not independent and therefore aggregate variance depends also on the covariance among individual decisions, as dictated by the network structure, so that

$$\text{var}\left(\sum_i k_i\right) = \sum_i \text{var}(k_i) + \sum_{i \neq j} \text{cov}(k_i, k_j) \ . \tag{19}$$

The impact of social learning is twofold. First, as shown in Proposition 1, the variance of individual decisions is lower, implying that the first term in Eq. (19) is lower when compared to the case of isolated firms in Eq. (18). Second, social learning introduces a covariance element given by the second term in Eq. (19). Proposition 2 shows that the net effect depends on the structure of the observational network and on the weights attached by each firm to the different sources of information.

In particular, if the vector v has heterogeneous entries, aggregate volatility increases. The only case in which the variance of aggregate investment under social learning is equal to the case of isolation is when the signal of each firm in the economy has exactly the same influence on aggregate investment, i.e., when $v_j = 1$ for all j. This scenario is verified when the network is regular, i.e., all firms have the same weighted in-degree and out-degree in the observational network, as in Fig. 1(b), and the weights α_i are homogeneous. Any other case results in an influence vector with heterogeneous elements and thus aggregate volatility increases with respect to the case of isolation.

The following example illustrates the impact of heterogeneous centrality among firms.

Example 3.

Consider the two economies described in Figs. 1(a) and 1(b) and the optimal weights derived in Examples 1 and 2. Given the matrices C_{star} and $\mathrm{C}_{regular}$ computed in Examples 1 and 2, we can compute the influence vectors associated to each network using Eq. (14):

$$v_{star} = \begin{pmatrix} 2 \\ 2 \\ 1/3 \\ 1/3 \\ 1/3 \end{pmatrix} \qquad v_{regular} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} .$$

The influence vector is given by the sum over the columns of the matrix C, meaning that each element v_i is sum of the weights given by each firm to the signal s_i. The higher is v_i, the higher the *centrality* of firm i. In the star network, firm 1 is highly central since all firms look at its decision when making their own choices. Note that firm 2 is also central. The reason is that the decision of firm 2 is used by firm 1. The centrality of each firm i, as measured by the influence vector, depends not only on the number of firms looking at firm i, but also on the number of firms looking at the firms who are looking at firm i. This means that the influence of a firm is recursively related to the influence of the firms who observe its decision (see Jackson et al., 2015). On the contrary, in the regular network all firms have the same centrality. Using Eq. (15) we can compute the variance of aggregate investment for both networks

$$\operatorname{var}(K_{star}) = 0.33\sigma^2 \qquad \operatorname{var}(K_{regular}) = 0.20\sigma^2 \ .$$

In the case of regular network aggregate variance is equal to the variance in isolation, i.e., $\sigma^2/5$, meaning that the reduction in the sum of individual variances and the positive covariances in individual decisions balance each other out (see Eq. (19)). On the opposite, heterogeneity in the centrality of firms, as in the case of the star network, leads to an increase in aggregate volatility. ♣

3.3 Decay Rate of Aggregate Volatility

The possibility of significant aggregate fluctuations originating at the micro level, i.e., at the firm or sector level, is often discarded in macroeconomics following a diversification argument. According to the latter, in an economy composed by N firms, the impact of independent idiosyncratic shocks on the fluctuations of aggregate investment should be negligible since, as $N \to \infty$, firm-level disturbances should decay at a rate $1/\sqrt{N}$. Gabaix (2011) proves that the $1/\sqrt{N}$ diversification argument does not apply when the firm size distribution is sufficiently fat-tailed, while Acemoglu et al. (2012) show that the argument is not valid in the presence of asymmetric input-output links between sectors.

In the following we show that, even in the presence of firms with identical size and without input-output interconnections between different sectors, the diversification argument may not hold when information is dispersed and firms try to reduce their uncertainty by learning from other firms' decisions.

Consider a sequence of economies indexed by the number of firms $N \geq 1$, with the network of informational links in each economy denoted by W_N. The corresponding sequences of aggregate investment and influence vectors are denoted respectively by

$\{K_N\}$ and $\{v_N\}$. Assuming that the variance of idiosyncratic signals is independent of the size of economy N, e.g., $\sigma_N^2 = \sigma^2 \ \forall N$, we have that

$$\text{std}\left(K_N\right) = \sqrt{\text{var}\left(K_N\right)} = \frac{\sigma}{N}\sqrt{\sum_j v_{j,N}^2} \ .$$

Let us define the following notation. Given two series of positive real numbers $\{x_N\}$ and $\{a_N\}$, we write $x_N \sim a_N$ if there exist $M \geq 0$ and finite constants $A > 0$ and $A \leq B < \infty$, such that $\inf_{N \geq M} x_N/a_N \geq A$ and $\sup_{N \geq M} x_N/a_N \leq B$. In other words, $x_N \sim a_N$ means that for $N \geq M$ the sequences $\{x_N\}$ and $\{a_N\}$ grow at the same rate. Moreover, we write $x_N = \Omega(a_N)$ if there exist $M \geq 0$ and finite constant $A > 0$, such that $\inf_{N \geq M} x_N/a_N \geq A$. This means that, for decreasing sequences, x_N decreases more slowly than a_N.

Therefore we can write that

$$\text{std}\left(K_N\right) \sim \frac{1}{N}\left\|v_N\right\|_2 \ . \tag{20}$$

Eq. (20) implies that the volatility of aggregate investment may decay with a rate different from $1/\sqrt{N}$, according to the properties of the social network summarized by vector v. The limiting behavior of $\left\|v_N\right\|_2$ as $N \to \infty$ depends on the distribution of $v_{j,N}$. In the following proposition we characterize the decay rate of aggregate volatility for both cases of fat-tailed and thin-tailed distribution of $v_{j,N}$.

Proposition 3. *Consider a series of economic systems indexed by $N \geq 1$. Assume that the sequence of influence vectors $v_1, \ldots, v_N$ has a power law distribution*

$$P_N\left(v_{j,N} > x\right) = c_N L(x) x^{-\zeta}$$

where $c_N \sim 1$ is a sequence of real positive numbers, $L(x)$ is slowly varying function, meaning that $\lim_{N\to\infty} L(x)x^{\epsilon} = \infty$ and $\lim_{N\to\infty} L(x)x^{-\epsilon} = 0$ for all $\epsilon > 0$, and $\max_j v_{j,N} \sim N^{1/\zeta}$. Then, as $N \to \infty$ aggregate volatility follows

a) $\text{std}\left(K_N\right) = \Omega\left(N^{(1-\zeta)/\zeta - \epsilon'}\right)$ *for* $1 < \zeta \leq 2$

b) $\text{std}\left(K_N\right) \sim N^{-1/2}$ *for* $\zeta > 2$

where $\epsilon' = \epsilon/(2\zeta)$.

The proof is in Appendix A. Proposition 3 implies that when the distribution of firms signals' influence v has thin tails ($\zeta > 2$), then the variance of aggregate investment decays at rate $1/\sqrt{N}$. On the contrary, when the distribution has fat tails ($1 < \zeta \leq 2$) the decay rate is much slower.

Therefore social learning may represent an additional reason for the failure of the diversification argument, depending on the topology of the informational network. In particular, a fat-tailed distribution of v implies a greater heterogeneity in the influences of firms' signals, corresponding to the case in which many firms look at the decision of the same small number of firms. The latter group of firms have high centrality in the network and therefore the influence of their signals decays slowly as the number of firms N increases. This implies that, in the presence of dispersed information, social learning and very asymmetric network structures, significant aggregate fluctuations may result from micro-level disturbances, even if the number of firms is very large.

4 Conclusions

The behavior of peers, friends or in general other members of a social or economic group, represents a valuable source of information for the *homo oeconomicus*. Observation of others' behavior is deeply rooted in human nature as a consequence of the adaptation to complex environments, where it is difficult to collect and process all available information. This paper shows that this micro-behavior, which we have called social learning, can have relevant consequences at the aggregate level. The aggregate effect of social learning depends on the topology of the network describing the links between agents. We show that, according to the network structure, social learning in the presence of dispersed information can explain, at least in part, aggregate fluctuations.

If the network is homogeneous, in the sense that all firms have the same influence in equilibrium, then aggregate volatility with social learning is at its minimum level and coincides with aggregate volatility with isolated firms. For any other network configuration, social learning leads to an increase in aggregate volatility. Aggregate variance is positively related to the concentration of influence in the network.

Moreover, we show that the diversification argument does not always apply in the presence of social learning. If the influence vector is sufficiently asymmetric, i.e., there exist few very influential firms in the network, then the aggregate impact of independent firm-level shocks does not decay at a rate equal to $1/\sqrt{N}$, resulting therefore in significant aggregate fluctuations. This result complements the findings of Gabaix (2011), who shows that the diversification argument does not hold when firms' size distribution is sufficiently fat-tailed, and Acemoglu et al. (2012) who show that the law of large number argument breaks down in the presence of asymmetric input-output links between productive sectors.

An important question regards the empirical relevance of social learning. From a qualitatively point of view, the impact of social learning crucially depends, as argued above, on the topology of the observational network. Galeotti and Goyal (2010) list a series of empirical works suggesting asymmetric topologies in informational networks. In accordance to the empirical evidence, they propose a network formation game, in which asymmetric topologies emerge as the equilibrium outcome. Moreover, Goyal et al. (2016) present experimental evidence supporting the emergence of asymmetric informational networks. Bikhchandani et al. (1992, 1998) argue in favor of the presence of *fashion leaders*, i.e., "expert" agents observed by many other agents, and Gilbert and Lieberman (1987) show that "smaller firms tend to imitate the investment activity of others". Therefore, we conclude that it is highly plausible that observational networks are asymmetric, and consequently that dispersed information and social learning play an important role in generating aggregate fluctuations.

Appendix A Proofs

A.1 Proof of Lemma 1

Proof. Define $\mathrm{A} \equiv (\mathrm{I} - \mathrm{D})\mathrm{W}$. Matrix $[\mathrm{I} - \mathrm{A}]$ is invertible when the spectral radius of A, defined as $\rho(\mathrm{A}) = \max_{1 \leq i \leq N} |\lambda_i|$ where λ_i is the i-th eigenvalue of A, is strictly smaller than one. Then notice that

$$\|\mathrm{A}\|_\infty = \max\left\{\sum_j |(1-\alpha_i)w_{ij}| \mid 1 \leq i \leq N\right\} = \max\left\{|(1-\alpha_i)| \mid 1 \leq i \leq N\right\} \ ,$$

given that matrix W is stochastic. Moreover, for a generic eigenvector-eigenvalue pair (x, λ) with $x \neq 0$, we have that $\lambda x = \mathrm{A}x$ and therefore

$$\|\lambda x\|_\infty = |\lambda| \, \|x\|_\infty = \|\mathrm{A}x\|_\infty \leq \|\mathrm{A}\|_\infty \|x\|_\infty \ \Rightarrow |\lambda| \leq \|\mathrm{A}\|_\infty \ ,$$

where the inequality follows from the submultiplicativity property of the matrix norm. When $0 < \alpha_i \leq 1 \ \forall i \in [1, N]$, we have that $\|\mathrm{A}\|_\infty < 1$ implying that $\rho(\mathrm{A}) < 1$. □

In the following we show that the equilibrium in Eq. (7) exists also under more general conditions. Before identifying such conditions let us introduce the following definitions. Following Golub and Jackson (2010), we define a group of nodes $Z \subset N$ as *closed* relative to a generic adjacency matrix Ω if $i \in Z$ and $\omega_{ij} > 0$ imply that $j \in Z$. A closed group of nodes is *minimally closed* relative to Ω if it is closed and no nonempty strict subset is closed.

The equilibrium in Eq. (7) exists when

a) $0 \leq \alpha_i \leq 1 \ \forall i \in [1, N]$ *with at least one* $\alpha_i > 0$, *and matrix* W *is irreducible.*

b) $0 \leq \alpha_i \leq 1$ and for each minimally closed group $\mathcal{Z}_j$ relative to $(\mathrm{I} - \mathrm{D})\mathrm{W}$ there exists at least one $i \in \mathcal{Z}_j$ such that $\alpha_i > 0$.

Proof. We start by proving that if $\mathrm{A} = (\mathrm{I} - \mathrm{D})\mathrm{W}$ is irreducible and at least one $\alpha_i > 0$, then it must be that $\rho(\mathrm{A}) < 1$. Notice that when at least one $\alpha_i > 0$, then matrix A is substochastic. Denoting by $e' = [1 \dots 1]$, this implies that $\mathrm{A}e \leq e$ and $\mathrm{A}e \neq e$. When matrix A is irreducible, from the Perron-Frobenius theorem it follows that $\rho(\mathrm{A}) = 1$ would imply $\mathrm{A}e = e$, which is impossible by construction. Therefore $\rho(\mathrm{A}) < 1$ follows from the result $\rho(\mathrm{A}) \leq \|\mathrm{A}\|_\infty$ derived above.

Let's now consider the case in which A is reducible. In general, if $0 \leq \alpha_i < 1$, the reducibility of matrix A follows from the reducibility of matrix W. If instead $\alpha_i = 1$ for some agents $i \in [1, N]$, then matrix A is reducible even if matrix W is irreducible. In what follows we define conditions such that $\rho(\mathrm{A}) < 1$ when A is reducible. If A is reducible, then following Meyer (2000, page 694), it is possible to write matrix A in the canonical form for reducible matrices

$$\mathrm{A} \sim \left(\begin{array}{cccc|cccc} \mathrm{A}_{11} & \mathrm{A}_{12} & \cdots & \mathrm{A}_{1r} & \mathrm{A}_{1,r+1} & \mathrm{A}_{1,r+2} & \cdots & \mathrm{A}_{1,m} \\ 0 & \mathrm{A}_{22} & \cdots & \mathrm{A}_{2r} & \mathrm{A}_{2,r+1} & \mathrm{A}_{2,r+2} & \cdots & \mathrm{A}_{2,m} \\ \vdots & & \ddots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \mathrm{A}_{rr} & \mathrm{A}_{r,r+1} & \mathrm{A}_{r,r+2} & \cdots & \mathrm{A}_{r,m} \\ \hline 0 & 0 & \cdots & 0 & \mathrm{A}_{r+1,r+1} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & \mathrm{A}_{r+2,r+2} & \cdots & 0 \\ \vdots & & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \mathrm{A}_{m,m} \end{array}\right),$$

where each $\mathrm{A}_{11}, \ldots, \mathrm{A}_{rr}$ is either irreducible or $[0]_{1\times 1}$, and $\mathrm{A}_{r+1,r+1}, \ldots \mathrm{A}_{mm}$ are irreducible. As noted in Meyer (2000), the effect of such a symmetric permutation is simply to relabel the nodes in the original network. Therefore a generic A_{sz} denotes the sub-network describing the connections from agents in rows s to agents in columns z. Define $\alpha(s)$ as the vector containing the α_i set by each agent i belonging to the set described by rows s.

As a first step, consider the matrices A_{kk} for $k = 1, 2, \ldots, r$ and observe that $\rho(\mathrm{A}_{kk}) < 1$ for each $k = 1, 2, \ldots, r$, for any possible value of entry α_i in vector $\alpha(k)$. This is certainly true when $\mathrm{A}_{kk} = [0]_{1\times 1}$, so consider the case in which A_{kk} is irreducible and notice that A_{kk} is substochastic by construction because there must be blocks A_{kj}, $j \neq k$ that have nonzero entries. From the previous result we know that irreducible substochastic matrices are characterized by a spectral radius strictly smaller than one.

Consider now the matrices A_{kk} for $k = r+1, \ldots, m$, which refer to the minimally closed groups relative to A. These matrices are substochastic if and only if at least one α_i in vector $\alpha(k)$ is positive. Once again, these irreducible stochastic matrices are characterized by a spectral radius strictly smaller than one.

Therefore, when at least one agent i in each minimally closed group of A sets $\alpha_i > 0$, we conclude that $\rho(\mathrm{A}) < 1$. □

In other words, for equilibrium in Eq. (7) to exist, each agent must receive at least one signal directly and/or indirectly. If $\alpha_i > 0 \ \forall i$, this condition is satisfied for any possible network topology. If instead $\alpha_i = 0$ for some agent i, then for equilibrium in Eq. (7) to exist, at least one agent i in any minimally closed group must have $\alpha_i > 0$ guaranteeing that all agents in the network are reached by at least one signal.

A.2 Proof of Proposition 1

Proof. Before proceeding with the proof of Proposition 1, we describe an important property of the matrix $\mathrm{C} \equiv [\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}]^{-1}\mathrm{D}$, which maps private signals into equilibrium investment decisions according to Eq. (7), in the following lemma.

Lemma 2. *Matrix C is a stochastic matrix, i.e.,* $\sum_j \hat{w}_{ij}\alpha_j = 1 \ \forall i$.

Proof of Lemma 2. Define the vector $e' = [1 \dots 1]$. Proving that $\sum_j \hat{w}_{ij}\alpha_j = 1 \ \forall i$ is equivalent to prove that $\mathrm{C}e = e$ or equivalently that $\mathrm{C}^{-1}e = e$, since C is invertible. Start from

$$\mathrm{C} = [\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}]^{-1}\mathrm{D} \ ,$$

and pre-multiply both sides by C^{-1} to get

$$\mathrm{I} = \mathrm{C}^{-1}[\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}]^{-1}\mathrm{D} \ .$$

Post-multiplying by D^{-1}

$$\mathrm{D}^{-1} = \mathrm{C}^{-1}[\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}]^{-1} \ ,$$

and by $[\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}]$ we get

$$\mathrm{D}^{-1}[\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}] = \mathrm{C}^{-1} \ .$$

Post-multiplying both sides by e, we have

$$
\begin{aligned}
\mathrm{C}^{-1}e &= \mathrm{D}^{-1}[\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}]e \\
\mathrm{C}^{-1}e &= \mathrm{D}^{-1}[e - (\mathrm{I} - \mathrm{D})\mathrm{W}e] \\
\mathrm{C}^{-1}e &= \mathrm{D}^{-1}[e - (\mathrm{I} - \mathrm{D})e] \\
\mathrm{C}^{-1}e &= \mathrm{D}^{-1}[e - e + \mathrm{D}e] \\
\mathrm{C}^{-1}e &= \mathrm{D}^{-1}\mathrm{D}e \\
\mathrm{C}^{-1}e &= e\,,
\end{aligned}
$$

where the third equality follows from the fact that W is stochastic. □

Having established the result in Lemma 2 we can proceed to prove Proposition 1 as follows. Eq. (12) follows from the comparison of Eqs. (10) and (11). We start by defining an M-matrix (Plemmons, 1977):

Definition. *An $N \times N$ matrix C that can be expressed in the form $C = sI - A$, where $a_{ij} \geq 0$ is the (ij)-th element of matrix A, $1 \leq i, j \leq N$ and $s \geq \rho(A)$, the maximum of the moduli of the eigenvalues of A, is called an M-matrix.*

It is straightforward to show that matrix $\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}$ is an M-matrix. Define $s = 1$ and $\mathrm{A} = (\mathrm{I} - \mathrm{D})\mathrm{W}$. By construction we know that $a_{ij} \geq 0$, while we showed in the proof of Theorem 1 that $\rho(\mathrm{A}) \leq 1$.

Since $\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}$ is an M-matrix, we know that it is inverse-positive (Plemmons, 1977), i.e., each element $\hat{w}_{ij}$ of $\hat{\mathrm{W}} = [\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}]^{-1}$ is non-negative. From Lemma 2 we know that $\sum_j \hat{w}_{ij}\alpha_j = 1$ and therefore, given that $0 < \alpha_j \leq 1\ \forall j$, we have that $0 \leq \hat{w}_{ij}\alpha_j < 1\ \forall j$. Therefore, defining $f(x) = x^2$, we have that

$$
\sum_j f(\hat{w}_{ij}\alpha_j) \leq f\left(\sum_j \hat{w}_{ij}\alpha_j\right) = 1\ ,
$$

where the inequality follows from the fact that f is a superadditive function for non-negative real numbers. The only case in which the above expression holds as an equality is when there is no social learning, i.e., when W is a zero matrix (meaning that $\hat{\mathrm{W}}$ is an identity matrix) and $\alpha_j = 1\ \forall j$. □

A.3 Proof of Proposition 2

Proof. Eq. (17) follows from the comparison of Eqs. (15) and (16). We can rewrite Eq. (17) as

$$\sum_j v_j^2 \geq N \Rightarrow \|v\|_2 \geq \sqrt{N} \; ,$$

and prove it using the Cauchy-Schwarz inequality. In fact, noticing from the results in Lemma 2 that $\sum_j v_j = N$, we can write

$$\begin{aligned} \left(\sum_j v_j^2\right) \cdot N &\geq \left(\sum_j v_j\right)^2 \\ \|v\|_2 \sqrt{N} &\geq N \\ \|v\|_2 &\geq \sqrt{N} \; . \end{aligned}$$

□

A.4 Proof of Proposition 3

Proof. We start by considering case *a)* in which $1 < \zeta \leq 2$. Recall from Eq. (20) that

$$\text{std}\left(K_N\right) \sim \frac{1}{N} \|v_N\|_2 \; .$$

For $N \to \infty$ we have that

$$\|v_N\|_2 = \sqrt{\sum_j v_{j,N}^2} = \sqrt{N \mathrm{E}\left[v_{j,N}^2\right]} \; .$$

Defining $v_{\max,N} \equiv \max_j v_{j,N}$ we can write, since v^2 is a non-negative random variable, that

$$\mathrm{E}\left[v_{j,N}^2\right] = \int_0^{v_{\max,N}} 2x P_N(v_{j,N} > x) dx = 2c_N \int_0^{v_{\max,N}} x L(x) x^{-\zeta} dx \; .$$

Given that $L(x)$ is a slowly varying function such that $\lim_{x\to\infty} L(x)x^{-\epsilon} = 0$ for $\epsilon > 0$, we have that

$$\mathrm{E}\left[v_{j,N}^2\right] = 2c_N \int_0^{v_{\max,N}} x L(x) x^{-\zeta} dx \geq 2c_N \int_0^{v_{\max,N}} x^{1-\zeta-\epsilon} dx \; .$$

Given that $v_{\max,N} \sim N^{1/\zeta}$, we can compute the integral to get

$$\mathrm{E}\left[v_{j,N}^2\right] \geq 2c_N(2-\zeta-\epsilon)^{-1}\left[x^{2-\zeta-\epsilon}\right]_0^{N^{1/\zeta}} = \bar{c}_N N^{\frac{2-\zeta-\epsilon}{\zeta}} \ ,$$

where $\bar{c}_N \equiv 2c_N(2-\zeta-\epsilon)^{-1}$. This implies that

$$\frac{1}{N}\left\|v_N\right\|_2 = \frac{1}{N}\sqrt{N\mathrm{E}\left[v_{j,N}^2\right]} \geq \bar{c}_N N^{\frac{1-\zeta}{\zeta}-\frac{\epsilon}{2\zeta}} \ .$$

From the last equation it follows that

$$\mathrm{std}\left(K_N\right) = \Omega(N^{\frac{1-\zeta}{\zeta}-\epsilon'}) \ ,$$

where $\epsilon' \equiv \frac{\epsilon}{2\zeta}$.

We now consider case *b)*. We start by noticing that when $\zeta > 2$ we have that $\mathrm{E}\left[v_{j,N}^2\right] = V$, where $0 < V < \infty$. Therefore

$$\left\|v_N\right\|_2 = \sqrt{N\mathrm{E}\left[v_{j,N}^2\right]} = \sqrt{NV} \ ,$$

from which it follows that

$$\mathrm{std}\left(K_N\right) \sim N^{-1/2} \ .$$

□

Appendix B Fixed Point Equilibrium

The equilibrium in Eq. (7) can be thought of as the fixed point of the dynamic process

$$k_\tau = \mathrm{D}s + (\mathrm{I}-\mathrm{D})\mathrm{W}k_{\tau-1} \ , \tag{21}$$

occurring in *notional* time τ, in which each firm observes the actions of other firms in the social network and updates its beliefs according to Eq. (6).

Iterating Eq. (21) we get

$$k_\tau = \sum_{z=1}^{\tau-1}\left[(\mathrm{I}-\mathrm{D})\mathrm{W}\right]^z \mathrm{D}s + \left[(\mathrm{I}-\mathrm{D})\mathrm{W}\right]^\tau k_0 \ .$$

Defining $\mathrm{A} \equiv (\mathrm{I} - \mathrm{D})\mathrm{W}$, when $\rho(\mathrm{A}) < 1$, i.e., when conditions a′), b′) and c′) are satisfied, we have that $\lim_{\tau \to \infty} \mathrm{A}^\tau = 0$ and $\sum_{z=0}^{\infty} \mathrm{A}^z = [\mathrm{I} - \mathrm{A}]^{-1}$. Therefore in equilibrium

$$k = [\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}]^{-1}\mathrm{D}s \ .$$

Appendix C Lucas-Phelps Island Economy

In this appendix we analyze the effect of social learning on individual pricing decisions and aggregate output in a Lucas-Phelps island economy, as in Morris and Shin (2002). Differently from Morris and Shin (2002), we do not have public information in the model, therefore we do not focus on the effect of the precision of public information but on the impact of social learning.

Consider an economy composed by N islands. Supply y_i^s in island i of a single consumption good is given by

$$y_i^s = bp_i \ ,$$

where $b > 0$ and p_i is the price chosen in island i. The demand y_i^d on island i is given by

$$y_i^d = c\left(\theta - p_i\right) \ ,$$

where θ is the money supply and $c > 0$. Market clearing implies that the optimal price decision under perfect information is

$$p_i = \frac{c}{b+c}\theta \ .$$

When θ is unknown, each firm forms expectations on the money supply θ and sets the price

$$p_i = \frac{c}{b+c}E_i\left(\theta\right) \ .$$

Assume that firms receive a private signal on the money supply as described in Eq. (3), and that θ is drawn from an improper uniform distribution. In isolation the expected money supply is

$$E_i(\theta) = s_i \ ,$$

and the optimal price is

$$p_i = \frac{c}{b+c}s_i \ .$$

When firms are able to observe the price decision of a subset of firms in other islands, they will use this information when setting their price. Similarly to Eq. (6), the price decision in the presence of social learning is:

$$p_i = \alpha_i \beta s_i + (1 - \alpha_i) \sum_j w_{ij} p_j \ ,$$

where α_i and w_{ij} have the same interpretation as in Section 2, while $\beta \equiv c/(b+c)$. The equilibrium price decision in all islands can be written as

$$p = \beta [\mathrm{I} - (\mathrm{I} - \mathrm{D})\mathrm{W}]^{-1} \mathrm{D} s \ , \tag{22}$$

where W is the adjacency matrix describing the network of social learning and D is a diagonal matrix such that $[\mathrm{D}]_{ii} = \alpha_i \ \forall i$. Firms sell at price p_i described in Eq. (22), and equilibrium production is determined by market clearing. Production on island i is computed using the demand function determined by money supply θ and price p_i:

$$y_i = c\left(\theta - p_i\right) \ . \tag{23}$$

From Eq. (23) it is clear that the variance of individual production is proportional to the variance of price decision p_i. From Proposition 1, we know that observing the price decisions of other firms reduces the volatility of individual actions, and it is therefore beneficial to firms. Aggregate output, i.e., the GDP of the island economy, is the sum of the individual production decisions:

$$Y = \sum_i y_i = cN\theta - c \sum_i p_i \ .$$

From Proposition 2 we know that the variance of aggregate production normalized by N depends on the topology of the observational network. If the network is asymmetric, aggregate volatility is higher relative to the same economy in which firms take their price decisions in isolation. Finally, when the observational network is characterized by the properties in Proposition 3, the variance of normalized aggregate output does not decay following the law of large numbers.

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